

## Proposed Projects for the course 2021/22

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## Course and research projects for students of 236331

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- Course projects 1-10 are for Fall Semester 2021/22.
- Thesis projects are for graduate students looking for a research topic. This list is still to be expanded.
- More items can be added if suggested.
- Course projects 4-6, 8-10 were chosen by the current students, see the registration list

## Course Project 1: More on definability of hereditary and monotone graph properties

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In the paper:

Syntax vs. Semantics on Finite Structures

Natasha Alechina and Yuri Gurevich

In: Structures in Logic and Computer Science, A Selection of Essays in Honor of Andrzej Ehrenfeucht. 1997. 14-33.

the authors show:

- There is a hereditary property of finite relational structures, which is definable by a formula in FOL, but is **not definable by a universal formula**.

Can we make this work for graphs or hypergraphs?

- Many more definability theorems for properties of relational structures closed under homomorphisms etc.

Can we characterize the definability for monotone graph properties?

## Course Project 2: MSOL and the polynomial hierarchy

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In the paper J.A. Makowsky and J.P. Marino.

"Tree-width and the monadic quantifier hierarchy."

Theoretical computer science 303.1 (2003): 157-170.

the authors discuss the complexity of checking whether an MSOL-formula is true in a graph.

Present their theorem:

For every level  $\Sigma_k^{poly}$  of the polynomial hierarchy **PH** there is a graph property which is MSOL-definable and complete for  $\Sigma_k^{poly}$ .

## Course Project 3: Where FOL and MSOL coincide

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The authors study on which classes of graphs FOL and MSOL have the same expressive power.

- Elberfeld, Michael, Martin Grohe, and Till Tantau.  
"Where first-order and monadic second-order logic coincide."  
ACM Transactions on Computational Logic (TOCL) 17.4 (2016): 1-18.

While reading the paper there are several questions which could be addressed further:

- What can we say when we include CMSOL into the picture?
- How can we extend the result to general relational structures?

## Course Project 4: Mice in labyrinths

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This is about the power of multi-head automata and automata with pebbles. If mice were finite automata, they would get lost in a corn field (labyrinth). If mice use a few pebbles they can find their way out, though. This is Budach's Theorem, as described in Hemmerling's book. It had various incomplete proofs. A more transparent and complete proof was given by Blum and Kozen. If pebbles are replaced by multiple heads, similar results are true. The number of heads gives a proper hierarchy, as shown by Yao and Rivest, and also by Ibarra.

The project consist in preparing a two hour lecture for students, summarizing these results with at least one substantial complete proof.

- Hemmerling, Armin. Labyrinth problems: Labyrinth-searching abilities of automata. Vol. 114. Springer-Verlag, 2013.
- Blum, Manuel, and Dexter Kozen. "On the power of the compass (or, why mazes are easier to search than graphs)." a 19th Annual Symposium on Foundations of Computer Science (sfcs 1978). IEEE Computer Society, 1978.
- Yao, Andrew C., and Ronald L. Rivest. " $k+1$  heads are better than  $k$ ." Journal of the ACM (JACM) 25.2 (1978): 337-340.
- Ibarra, Oscar H. "On two-way multihead automata." Journal of Computer and System Sciences 7.1 (1973): 28-36.

## Course Project 5: Complexity of EF-Games

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Given two finite structures decide which player in the EF game wins. We study here the complexity of this problem.

- Pezzoli, Elena. "Computational complexity of Ehrenfeucht-Frass games on finite structures." International Workshop on Computer Science Logic. Springer, Berlin, Heidelberg, 1998.
- Khoussainov, Bakhadyr, and Jiamou Liu. "On complexity of EhrenfeuchtFrass games." Annals of Pure and Applied Logic 161.3 (2009): 404-415.

## Course Project 6: The Complexity of HEX

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HEX is graph decision problem which is not known to be definable in SOL. In this project we show that HEX is PSpace-complete. Variations on HEX are also studied and their complexity is to be discussed.

This is also a topic for further research (MSc and PhD).

- Even, Shimon, and Robert Endre Tarjan. "A combinatorial problem which is complete in polynomial space." *Journal of the ACM (JACM)* 23.4 (1976): 710-719.
- Arratia-Quesada, Argimiro A., and Iain A. Stewart. "Generalized Hex and logical characterizations of polynomial space." *Information processing letters* 63.3 (1997): 147-152.
- Arratia, Argimiro. "On the descriptive complexity of a simplified game of Hex." *Logic Journal of IGPL* 10.2 (2002): 105-122.

## Course Project 7: Model checking for SOL, MSOL, FOL

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Given  $\phi$  and  $\mathcal{A}$  the problem is to determine the complexity of  $\mathcal{A} \models \phi$ ?

If both  $\phi$  and  $\mathcal{A}$  are part of the input this is **PSpace**-complete for  $\phi \in \text{SOL}$ , but also for  $\phi \in \text{FOL}$  or any logic inbetween.

If  $\phi$  is fixed and only  $\mathcal{A}$  is the input this varies.

- Makowsky, Johann A., and J. P. Marino. "Tree-width and the monadic quantifier hierarchy." Theoretical computer science 303.1 (2003): 157-170.
- Makowsky, Johann A., and Yachin B. Pnueli. "Arity and alternation in second-order logic." Annals of Pure and Applied Logic 78.1-3 (1996): 189-202.
- Vardi, Moshe Y. "The complexity of relational query languages." Proceedings of the fourteenth annual ACM symposium on Theory of computing. 1982.
- For Vardi's result see any monograph on Finite Model Theory.

## Course Project 8: Easy cases of SAT: Linear time algorithms

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Present linear time algorithms for 2SAT, HornSAT, and renamable Horn.

- Even, Shimon, Alon Itai, and Adi Shamir. "On the complexity of time table and multi-commodity flow problems." 16th Annual Symposium on Foundations of Computer Science (sfcs 1975). IEEE, 1975.
- B Aspvall. Recognizing disguised  $nr(1)$  instances of the satisfiability problem. Journal of Algorithms, 1:97103, 1980
- del Val, Alvaro. "On 2-SAT and renamable Horn." In AAAI/IAAI, pp. 279-284. 2000.
- Itai, Alon, and Johann A. Makowsky. "Unification as a complexity measure for logic programming." The Journal of Logic Programming 4.2 (1987): 105-117.
- Dowling, William F., and Jean H. Gallier. "Linear-time algorithms for testing the satisfiability of propositional Horn formulae." The Journal of Logic Programming 1.3 (1984): 267-284.
- M. G. Scutella. A note on dowling and galliers top-down algorithm for propositional horn satisfiability. Journal of Logic Programming, 8:265273, 1990

## Course Project 9: Analyzing graph classes for definability

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Choose several graph classes from [www.graphclasses.org](http://www.graphclasses.org) and fill the table below

| Graph class | monotone | hereditary | minor closed | FOL-def | CMSOL-def | SOL-def |
|-------------|----------|------------|--------------|---------|-----------|---------|
| Forest      | yes      | yes        | yes          | no      | yes       | yes     |
| Planar      | yes      | yes        | yes          | no      | yes       | yes     |
| Chordal     | no       | yes        | no           | no      | yes       | yes     |
| Your Class  | ??       | ??         | ??           | ??      | ??        | ??      |

- If yes answer give the list of forbidden (subgraphs, induced subgraphs, minors), respectively the formula.
- If no answer give counterexample or prove non-definability using Hankel matrices.

## Course Project 10: Lower bounds for Feferman-Vaught Theorem

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Explain the lower bounds given in:

- Dawar, A., Grohe, M., Kreutzer, S., and Schweikardt, N.  
Model theory makes formulas large.  
In International Colloquium on Automata, Languages, and Programming  
(2007).  
pp. 913-924. Springer, Berlin, Heidelberg. .

## Research topic A: Definability and computability for matroids

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Develop a coherent theory of complexity for computations with matroid oracles.

This is problem 5 in Kotek-Makowsky below.

- Mayhew, Dillon.  
"Matroid complexity and nonsuccinct descriptions."  
SIAM Journal on Discrete Mathematics 22.2 (2008): 455-466.
- Kotek, Tomer, and Johann A. Makowsky.  
"The exact complexity of the Tutte polynomial."  
arXiv preprint arXiv:1910.08915 (2019).
- A shorter version of the above will appear in the Handbook on the Tutte polynomial and Related topics, CRC Press, 2022.
- Snook, Michael. "Matroids, Complexity and Computation." (2013).

## Research topic B: Is there a maximal logic satisfying the Feferman-Vaught Theorem?

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In the paper below we overstated the result. Can we rescue this approach?

- Labai, Nadia, and Johann A. Makowsky.  
"Logics of Finite Hankel Rank."  
Fields of Logic and Computation II. Springer, Cham, 2015. 237-252.  
also as: arXiv preprint arXiv:1512.02507 (2015).

## Research topic C: More applications of Hankel matrices

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- Gessel rank
- Specker rank (substitution rank)

## Research topic D: Patch-width of structures

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- Fischer, Eldar, and Johann A. Makowsky.  
"On spectra of sentences of monadic second order logic with counting." a  
The Journal of Symbolic Logic 69.3 (2004): 617-640.
- Shelah, Saharon, and Mor Doron.  
"Relational structures constructible by quantifier free definable operations."  
The Journal of Symbolic Logic 72.4 (2007): 1283-1298.
- Adler, Hans, and Isolde Adler.  
"A note on clique-width and tree-width for structures."  
arXiv preprint arXiv:0806.0103 (2008).
- Klazar, Martin. "Some general results in combinatorial enumeration."  
Permutation patterns 376 (2010): 3-40.  
also as "Overview of some general results in combinatorial enumeration."  
arXiv preprint arXiv:0803.4292 (2008).

## Research Project E: Computing permanents on special classes of matrices.

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- Barvinok, Alexander I. "Two algorithmic results for the traveling salesman problem." *Mathematics of Operations Research* 21.1 (1996): 65-84.
- Courcelle, Bruno, Johann A. Makowsky, and Udi Rotics. "Linear time solvable optimization problems on graphs of bounded clique-width." *Theory of Computing Systems* 33.2 (2000): 125-150.
- Kogan, Grigory. "Computing permanents over fields of characteristic 3: Where and why it becomes difficult." *Proceedings of 37th Conference on Foundations of Computer Science*. IEEE, 1996.
- Abbott, Tim, and Alex Schwendner. "The Complexity of the Permanent and Related Problems." (2007).