Model Theory in Computer Science: My Own Recurrent Themes.

(and some lessons I learned)

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Overview

- Mathematical Logic
- Categoricity, M. Morley and and S. Shelah
- Abstract Model Theory, Reduction and Interpolation Theorems
- Model theory in Computer Science
- Database Theory and Logic Programming
- From Courcelle's Theorem to Graph Polynomials
- Some lessons I learned, if time permits.

Logic vs Computer Science

In the 70ties and later there was a lot of

resistance and suspicion

againts the marriage of LOGIC and CS.

- From the EE-CS community
- From logic community like the ASL and especially DVMLG
- From the mathematical community on departemental level.

Model Theory vs Proof Theory

Logic is traditionally divided (well roughly speaking) into

- Recursion Theory aka Computability.

  Leads to Complexity Theory and Algorithms

- Model Theory aka Semantic reasoning.

  Leads to Automata, Data Bases, Finite Model Theory, Verfication, Model Checking, etc

- Proof Theory aka Syntactic reasoning.

  Leads to Automated Theorem proving, Linear Logic, Type Theory, etc

- Set Theory aka Modeling of mathematical objects.

  Leads to Category Theory, Specification of Data Types, Definition of Data Types, etc

Back to overview

Model Theory: Categoricity and Finite Axiomatizability

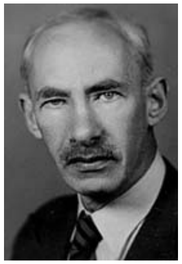
My first attempt to tackle open problems was a consequence of reading

M. Morley's fundamental paper on categoricity in power,

in the undergraduate seminar in mathematical logic at ETH Zurich,
held by E. Specker and H. Läuchli,
and regularly attended by the then still very lucid octogenarian P. Bernays.

Ernst Specker

Hans Läuchli



Paul Bernays

$\aleph_0$-Categoricity

A first order theory T is categorical in some infinite cardinal κ if T has no finite models and all its models of size κ are isomorphic.

Categoricity of First Order theories was one of the dominating topics of model theory since the late 1950ties.

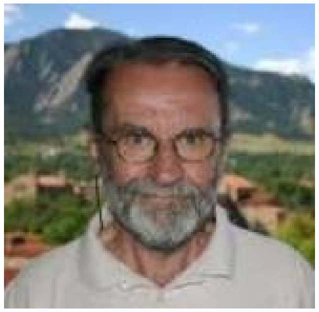
C. Ryll-Nardzewski

L. Svenonius

E. Engeler

They characterized, independently, the $\aleph_0$-categorical theories.

$\aleph_1$-Categoricity



A. Ehrenfeucht

A. Mostowski

R. Vaught

- A. Ehrenfeucht and A. Mostowski studied models with large automorphism groups, using Ramsey Theory.
- R. Vaught, together with M. Morley, applied this to $\aleph_1$-categorical theories.

Morley's Theorem (1965)

Michael Morley (1930-today)

Building on their work, Michael Morley proved in 1965 the first truly deep theorem of model theory:

> If Σ is κ-categorical for **some** uncountable κ
> then Σ is κ-categorical for **every** uncountable κ.

More importantly, even, the paper ends with a list of questions, which still today shape research in model theory.

Morley's Questions

Morley asks, among other questions:

Is there a finitely axiomatizable first order theory T which is

- κ-categorical for all κ, or
- for all uncountable κ.

Attacking these questions required
understanding of the structure theory of κ-categorical theories
(stable theories, rank, degree, etc.)

and

some idea on how
to prove or disprove finite axiomatizability.

How to prove non-finite axiomatizability?

I made a thorough manual literature search in the library about finite axiomatizability, (no scholar.google.com was available then).

From this I learned about

- Ehrenfeucht-Fraïssé games,
- ultraproducts, and
- other methods,

but only the **Ehrenfeucht-Fraïssé games** seemed **promising** to me.

My early success

With some ideas on how to approach Morley's question, I attended my **first logic conference in 1970**, where I received encouragement and a still unpublished preprint of from A. Lachlan.

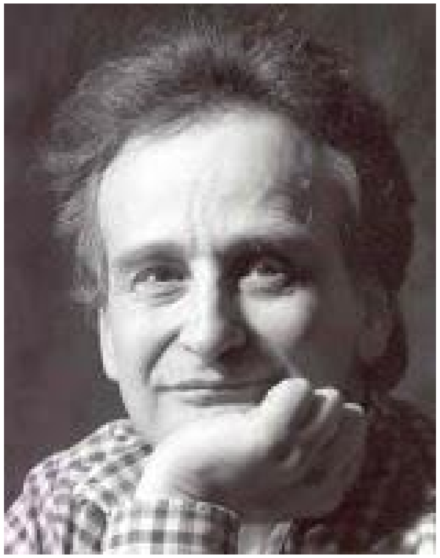
Upon my return I asked my supervisor, H. Läuchli, whether I could write my M.Sc. thesis about Morley's question, and he agreed.
I managed to prove in 1970

Theorem 1
(i) A first order theory T which is $\aleph_0$-categorical and strongly minimal (hence categorical in all infinite $kappa$) cannot be finitely axiomatizable.
(ii) There is a finitely axiomatizable complete first order theory T which is superstable.
(iii) If there is an **infinite, finitely presentable group with only finitely many conjugacy classes**, *there is also a finitely axiomatizable $\aleph_1$-categorical theory.*

It is still open today whether such a group exists.

Enter S. Shelah

Most of Morley's problems were solved by S. Shelah,



S. Shelah

Just the finite axiomatizability questions withstood his attacks.

The solutions

The finite axiomatizability questions were finally solved

- by M. Peretyatkin:
  there is a finitely axiomatizable ω_1-categorical theory,
- by G. Cherlin, L. Harrington and A. Lachlan and by B. Zil'ber:
  there is no finitely axiomatizable theory categorical in all infinite powers.
- G. Cherlin, L. Harrington and A. Lachlan use the **classification theory of finite groups**, and
- B. Zil'ber uses results on **diophantine equations**

to overcome the difficulties I had not been able to overcome.

Neither of these tools were **available** when I left the problem.

30 years later......

Thirty years later I discussed with Boris Zil'ber my work on

graph polynomials.

It turned out that, unknown to him, they are appear implicitly in his work on the structure of totally categorical theories.

This led us to the discovery of infinitely many new graph polynomials.

Generalized quantifiers and Abstract Model Theory

- It was Wiktor Marek, who introduced me in 1971 to

  Lindström's Theorem.

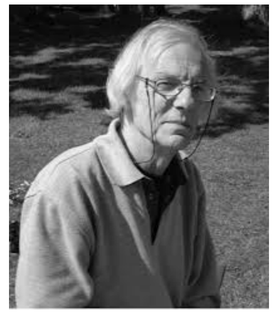
  It had been rediscovered by Harvey Friedman, who gave it much publicity.

- What a great Theorem:

> Predicate Logic can be characterized, among *all the logics* as the only one which satisfies the Löwenheim-Skolem Property and one of the following: compactness or axiomatizability.

- A new paradigm was found, which consisted in characterizing logics.

Generalized quantifiers and Abstract Model Theory



Per Lindström (1936-2009)

- I immediately studied Lindström's papers and all that was known about extensions of first order logic and prepared a seminar talk about it.
- Later P. Lindström told me that his original motivation for the theorem had been to find a new application of Ehrenfeucht-Fraïssé games.

Here they are again:

Ehrenfeucht-Fraïssé games.

I spent 1972 partially in Warsaw as an exchange student,
working under the late A. Mostowski on generalized quantifiers.

Beyond Lindström

- There were two lines of studying extensions of first order logic:
  (i) via generalized quantifiers, and
  (ii) via fragments of infinitary logics.

- J. Barwise showed that

> the admissible fragments of $\mathcal{L}_{\omega_1,\omega}$
> satisfy the **Craig Interpolation Theorem**.

- D. Scott and, independently before, E. Engeler, had shown that

> all countable structures over a countable vocabulary can be characterized up to isomorphism by a sentence in $\mathcal{L}_{\omega_1,\omega}$.

Georg Kreisel

, Georg Kreisel (1923-2015)

- What I tried to do:

  Find characterizations of logics
  using other properties than
  the Löwenheim-Skolem-Tarski Theorem and the Compactness Theorem.

- G. Kreisel suggested his own criteria of choosing logics, in his 1968 paper:
  The choice of infinitary languages

- In Fall 1972 E. Engeler and P. Bernays introduced me to G. Kreisel.

Skip Kreisel

Kreisel's help

- I told G. Kreisel about my ideas,
  and he got very interested and encouraging.

- An intensive correspondence followed which lasted till 1975.

- He also provided me with a preprint of

  L. Tharp,
  without telling me that he was refereeing it.

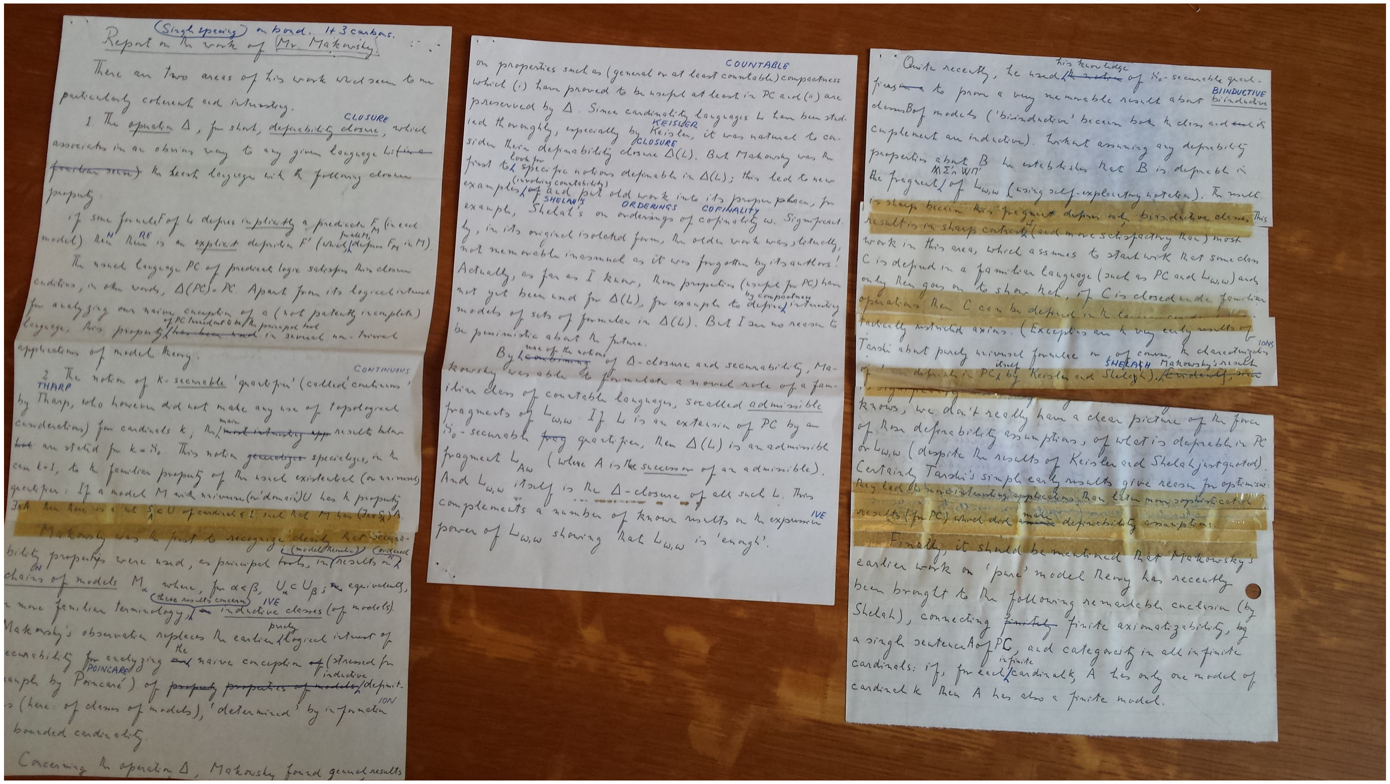
- I naively used the material in my PhD thesis,
  trusting that it was given to me in good faith for use.

- I generalized Tharp's definitions and proved innocently theorems which may have been also on Tharp's mind.

- In Spring 1973 he sent a telegram offering me a position at Stanford.

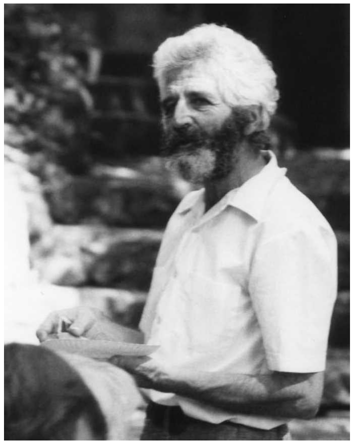
Kreisel's letter of recommendation

Kreisel later wrote very positive recommedation letters for me.
I found two of them accidentally while searching recently in the Kreisel Archive for his letters to Iris Murdoch.



Letter for Yale University, Spring 1974

Craig's Interpolation Theorem (1957)



William Craig (1918-2016)

UC Berkeley obituary

R. Zach's obituary of W. Craig

Craig's Interpolation Theorem

Let $\bar{R}, \bar{S}, \bar{T}$ be three disjoint finite relational vocabularies.

Let $\phi(\bar{R}, \bar{S})$ and $\psi(\bar{S}, \bar{T})$ two FOL-sentences such that

$$\models \phi(\bar{R}, \bar{S}) \rightarrow \psi(\bar{S}, \bar{T}),$$

then there is a FOLsentence $\theta(\bar{S})$ such that

$$\models \phi(\bar{R}, \bar{S}) \rightarrow \theta(\bar{S}) \text{ and } \models \theta(\bar{S}) \rightarrow \theta(\bar{S}, \bar{T})$$

Equivalently, if

$$\phi(\bar{R}, \bar{S}) \models \psi(\bar{S}, \bar{T}),$$

then there is a FOLsentence $\theta(\bar{S})$ such that

$$\phi(\bar{R}, \bar{S}) \models \theta(\bar{S}) \text{ and } \theta(\bar{S}) \models \theta(\bar{S}, \bar{T})$$

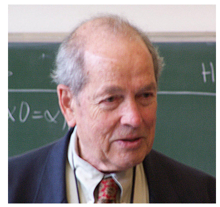
Characterizing $\mathcal{L}_{\omega_1,\omega}$

Having worked on categorical first order theories, I formulated and finally proved the following in 1973:

Theorem 2
Let $\mathcal{L}$ be a logic such that Craig's Interpolation Theorem holds for $\mathcal{L}$ and such that all countable structures over a countable vocabulary can be characterized up to isomorphism by a sentence in $\mathcal{L}$. Then $\mathcal{L}_{\omega_1,\omega}$ is a sublogic of $\mathcal{L}$.

For this theorem I was inspiried by three papers by S. Feferman, which among other things discuss abstract versions of the Feferman-Vaught Theorem, which entered my toolbox already then.

Reduction Theorems for Products and Sums

A. Mostowski 1913-1975 S. Feferman 1928-2016 R. Vaught 1926-2002

Theorem C:(Feferman and Vaught, 1959)

For every formula $\phi \in \mathsf{FOL}^q(\tau)$ one can compute a sequence of formulas

$$\langle \psi_1^A, \ldots \psi_m^A, \psi_1^B, \ldots \psi_m^B \rangle \in \mathsf{FOL}^q(\tau)^{2m}$$

and a Boolean function
$B_\phi : \{0,1\}^{2m} \rightarrow \{0,1\}$ such that

$$\mathfrak{A} \sqcup \mathfrak{B} \models \phi \text{ iff } B_\phi(b_1^A, \ldots b_m^A, b_1^B, \ldots b_m^B) = 1$$

where $b_j^A = 1$ iff $\mathfrak{A} \models \psi_j^A$ and $b_j^B = 1$ iff $\mathfrak{B} \models \psi_j^B$

Similarly for MSOL

S. Feferman in April 2016 at the dinner after the Tarski Lectures in Berkeley

S. Feferman passed away on July 26, 2016
Feferman's obituary

Recurrent themes so far

- Ehrenfeucht-Fraïssé games
- Reduction Theorems
- Interpolation Theorems
- Abstract Model Theory

The Promised Land: Logic and Computer Science

- Between 1975 and 1980 I was working in Berlin (FU) and in Israel (Hebrew University)

- Israel: Because I was collaborating with S. Shelah and increasingly with other Israeli logicians, mathematicians and emerging computer scientists.

- Israel: I got married to an Israeli

This had major impact on my career planning: I planned to move into Israeli Academia, and to do this I was advised to embrace Logic and its potential impact in Computer Science.

Thanks to S. Shelah and especially Eli Shamir, and later to Catriel Beeri, Shimon Even, Amir Pnueli and Dov Gabbay for their support.

Background in Theoretical Computer Science

I was well aware that mathematical logic, especially model theory, had something to offer to theoretical computer science.

- I had attended the Specker-Strassen Seminar in Zurich in the early 70ties, where we studied evolving complexity theory.

- I had attended E. Engeler's seminar in Zurich, where we studied how to use computers in combinatorial group theory.

- It was finally E. Shamir in Jerusalem who gave me the cruical impulse to approach computer science successfully. In 1978 he arranged for a "blind date" with C. Beeri, who was struggling to find the right definition of database dependencies.

- He also told me to attend the ACM-STOC conference in 1979 in Atlanta, where I got acquainted with V. Pratt and his dynamic logic.

How to use model theory in computer science?

- I tried to identify problems in theoretical computer science which could be tackled using model theoretic methods. I was looking for model theoretic characterizations of certain classes of syntactically defined formulas and for analogues of Lindström's Theorems.

- At the Logic Colloquium 1982 in Florence I was an invited speaker and I gave a talk on *Model theoretic issues in theoretical computer science: Relational Data Bases and Abstract Data Types*.

- Y. Gurevich discussed this paper with me at great length in the years 1982-84, and it inspired him to write his paper Logic and the challenge of computer science. But I had written my paper for the wrong audience:

- The Logicians were not interested in Computer Science, and the first LiCS conference was held only in 1986. The first CSL conference was held in 1987, and EACSL was founded in 1992.

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Database Theory

From 1978 on I tried use my model theory knowledge in database theory.

- I worked with Catriel Beeri on the right definition of database dependencies.
- I proved the first undecidability result in dependency theory.
- I prepared the classification of dependencies as special classes of First Order Horn formulas.
- Much of this was further developed on **M. Vardi's PhD thesis**.
- I did not attend DB-conferences personally.
- Recognition came late, but it did come.

Conceptual modelling

- With my student **V. Markowitz** I laid the logical foundations of Entity-Relationship modelling.
- We gave a complete characterization of ER-Databases in terms of the Relational model.
- I did not attend DB-conferences personally.
- Recognition for it came late, but it did come:

Reduction theorems, translation schemes and preservation theorems

- With my student Elena Ravve we tried further applications of model theory to database design.
- But J. Ullman had declared database design a dead end!
- We succeded in explaining BCNF (Boyced Codd Normal form) but had no impact.
- We succeded in applying reduction theorem (Feferman-Vaught Theorems), but had no killing applications.

Why Horn formulas ?

I also tried to understand why Horn formulas played a central role both in Database Theory and in Logic Programming.

- With A. Itai we were the first to give a **linear algorithm for Horn SAT**.

  This is usually credited to
  Dowling, W., and Gallier, J., (1984) "Linear-time algorithms for testing the satisfiability of propositional Horn formulae". Journal of Logic Programming, 3, 267-284
  who did it later but promoted it agressively.

- With E. Dahlhaus and A. Israeli we gave polynomial time Algorthm for **Interpolation for propositional Horn formulas**.

  This remained almost unnoticed, because it was done in 1984 (published in 1987 in a journal), and lacked promotion.

- Based in work with B. Mahr I finally gave a model theoretic **Characterization of First Order Horn formulas** which was relevant for Logic Programming.

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Model Theory finally finds its use:

The Feferman-Vaught Theorem

In
AUTHOR =J.A. Makowsky,
TITLE =Algorithmic uses of the Feferman-Vaught theorem,
JOURNAL = Annals of Pure and Applied Logic,
VOLUME = 126.1-3, PAGES = 159-213, YEAR =2004

I finally found the model theoretic tool with the most dramatic applications.

- Courcelle's theorem on tree-width.
- With my student U. Rotics and with B. Courcelle we extended this to clique with.

From Computer Science to Finite Combinatorics

- I extended this approach to graph polynomials.

- Inspired by work of E. Specker, I extended these methods to counting problems in combinatorics.

  In collaboration with my students I. Averbouch and T. Kotek, and with Eldar Fischer and E. Ravve.

- SERIES = Contemporary Mathematics,
  TITLE =Model Theoretic Methods in Finite Combinatorics,
  EDITOR = M. Grohe and J.A. Makowsky,
  VOLUME = 558, YEAR = 2011, PUBLISHER = American Mathematical Society,

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Computing permanents

- I first came across the problem of computing the permanent at Specker's 60th birthday conference in 1980.
- There Valiant presented the complexity classes $\mathbf{VP}$ and $\mathbf{VNP}$
- The permanent of an $(n \times n)$-matrix $A = (A_{i,j})$ is given as

$$per(A) = \sum_{s:[n]\to[n]} \prod_{i\in[n]} A_{i,s(i)}$$

- Computing the permanent of a $(0,1)$-matrix is hard:

  (i) $\sharp$-complete in the Turing model of computation;

  (ii) $\mathbf{VNP}$-complete in Valiants algebraic model of computation.

Computing permanents on special matrices

A. Barvinok, 1996 Let $\mathcal{M}_r$ be the set of real matrices of fixed rank r.
For every $A \in \mathcal{M}_r$ there is a polynomial time algorithm $\mathcal{A}_r$ which computes $per(A)$.

JAM, 1996 Let $\mathcal{T}_w$ be the set of adjacency matrices of graphs of tree-width at most w. For every $A \in \mathcal{T}_w$ there is a polynomial time algorithm $\mathcal{A}_w$ which computes $per(A)$.

The two theorems are incomparable.

My theorem was inspired by my work with G. Kogan.
I wanted to prove a theorem which Kogan could not prove.

Grigori Kogan, a tragic case


Grigori Kogan (1969–)

- He arrived from Gorky (today Nishny-Novgorod) in Fall 1995, and came with recommendation letters which praised him highly (e.g., by G.M. Adelson-Velsky).
- He was a stunning virtuoso in Combinatorial Linear Algebra, and specialised in computing permanents.
- Unfortunately, he was psychologically unstable, which became worse after he was confronted with a serious gap in his proof of $\mathbf{NP} = \mathbf{RP}$.
- Nevertheless, he proved several remarkable results, and opened new roads in computing the permanent.

Skip result

Kogan's Theorems

Let K be a Galois field $K = GF(3^q)$ (of characteristic 3).

Theorem I: (1995)
The permanent of a matrix A in $K^{(nxn)}$ can be computed in time $O(n^c)$ with $c < 10$ in the following cases:

(i) $AA^T = I_n$ (i.e., $rk(AA^T - I_n) = 0$).

(ii) $rk(AA^T - I_n) = 1$.

Theorem II: (1995)
For matrices with $rk(AA^T - I_n) \geq 2$ computing the permanent is as difficult as the general case of $rk(AA^T - I_n) = n$,
namely it is $\oplus_3\mathbf{P}$-complete.

These results were presented at FOCS'96,

written up by M. Kaminski and JAM.

Learning from my students

I have learned a lot from some of my students:

- With **U. Rotics** and **V. Markowitz** I have laid the relational foundations of the Entity-Relationship model in databases.

- **U. Rotics** got me interested in B. Courcelle's work which led our collaboration with **B. Courcelle** on various notions of width in graph theory.

- With **E. Ravve** I explored the various uses of interpretability in databases.

- Finally, **G. Kogan's work** led me into the study of graph polynomials, which I pursued for the last twenty years.

- I would like to thank
  I. Averbouch, B. Godlin, T. Kotek, N. Labai and **E. Ravve**.
  They were (and still are) my major collaborators in the ongoing projects on graph polynomials and Hankel matrices for graph parameters.

Thanks

Some lessons I have been tought in these years

Some mathematicians have written lessons which have inspired me.
My favorite is

- G.C. Rota
  10 Lessons I wish I had been taught
  Notices of the AMS, Volume 44, Number 1, 1997

I pick now a few of my own lessons.

Pitfalls of memory and credit

Lesson 1 *I bit of introspection always helps:*

- How did I choose my particular topic?
  Was I inspired by somebody's talk?
- With whom did I talk about my work, and when?
- Did I quote all relevant papers?

It never hurts being generous with credit !

Y. Gurevich's advice

Lesson 2 *Don't compete, be inspired !*

I once started a paper with:

X and Y proved A. We now prove a stronger result, namely B.

Y wrote to me: "why are you so competitive?"

I did not think I was. I asked how he would have written it.

Inspired by the result A of X and Y we prove B.

S. Shelah's advice

Lesson 3

Never let ideology or aesthetics
prevent *you from proving a theorem.*

What is a good question (after B. Neumann)

Hanna Neumann wrote a book on *Varieties of groups* (1967)
where she also studies a large number, say n, of finiteness conditions
and some of their 2^n satisfiability problems.

In my M.Sc.-thesis I had reduced a problem to the simultaneous satisfiability of three such condtions.

I met her husband, B. Neumann, at a conference in 1974, and asked him about it.

That's a stupid question! B. Neumann told me.
Why that???
because we have not the slightest idea, how to attack the problem.

Lesson 4 *Good questions test the limits of existing tools.*

Concurrent discoveries are the rule.

Our fight for priorities is rarely justified.

Scientific discoveries grow in the community like mushrooms:

Time and temperature have to be right.

Ideas also grow and reach the public when time is right.

If an idea is truly formulated by one researcher only, then he is really ahead of his time.

- R.L. Wilder, Mathematics as a Cultural System, Pergamon Press, 1981

Lesson 5 *However, one can be too early and also too late. Or one can choose unfortunate terminology.*

Research vs marketing oneself (V. Arnold)

- Should we stay at home (in the library) and think?
- Should we rather travel a lot, to communicate?
- Should we attend conferences mostly to advertize our work (ourselves)?

Lesson 6 *We should* **serve science** *more than science should serve our individual purpose.*

Overfunded research

Lesson 7 *Overfunded research is like heroin:*

It makes one **addicted**,

weakens the mind and

furthers prostitution.

(jam in the Jerusalem Post 19.4.85)

Thank you ALL

for organizing

and

attending this occasion

However, (I hope) I will not retire from

research, writing, and critical thought!