

#### Andrzej Mostowski November 1, 1913 - August 22, 1975

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Warsaw, October 2013



# The Legacy of Andrzej Mostowski

#### Johann A. Makowsky

Faculty of Computer Science, Technion - Israel Institute of Technology, Haifa, Israel janos@cs.technion.ac.il

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#### The teachers ...





Kasimierz Kuratowski 1896-1980 Alfred Tarski 1901-1983

# ... and their joint books

- K. Kuratowski and A. Mostowski
  Set Theory
  Warsaw 1952 in Polish, 1966 2nd edition in Polish, 1967 in English
- A. Tarski, A. Mostowski and R. Robinson Undecidable Theories Amsterdam 1959

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## An anomaly of the ordered pair

In the first edition of Set Theory the following exercise can be found:

Show that there are no non-empty sets A and B such that  $A \cap B = A \times B$  !

- If the cartesian product were defined by an abstraction principle in the Fregian sense, indeed no such sets could exist.
- However, with the definition of the ordered pair (by Kuratowski, or any other definition), such sets can be constructed by transfinite induction.

Conclusion: Defining the ordered pair always has side effects.

#### Translation schemes and interpretability

In the book Undecidable Theories undecidability is often proved by reductions to arithmetic via interpretations. These ideas are also used and further developed in

- A. Mostowski, On direct products of theories, Journal of Symbolic Logic, 17 (1952), pp. 1-31.
- S. Feferman and R. Vaught, The first order properties of algebraic systems, Fundamenta Mathematicae, 47, (1959) pp. 57-103.
- M.O. Rabin, A simple method for undecidability proofs and some applications, in: Logic, Methodology and Philosophy of Science II, 1965, pp. 58-68.
- J.A. Makowsky, Algorithmic uses of the Feferman-Vaught theorem, Annals of Pure and Applied Logic, 126.1-3 (2004), pp. 159-213.

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## Other early influences in Poland



From left to right:

- W. Sierpinski, 1882-1969, S. Mazurkiewicz, 1888-1945,
- J. Lukasiewicz, 1878-1956, S. Lesniewski, 1886-1939,
- A. Lindenbaum, 1904-1941(?)

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### Vienna 1936-1937



The period in Vienna left a deep impact on A. Mostowski.

His interest in Set Theory and Metamathematics has it origin here.

A. Mostowski would both do research and expository work in these fields.

Kurt Gödel, 1906-1978

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#### Zürich 1937-1938



H. Hopf, 1894-1971

G. Polya 1887-1985

P. Bernays, 1888-1977

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#### ... and even Physics.



Wolfgang Pauli, 1900-1958

## The true humanist scholar of the age of reason

A. Mostowski's introduction to science and scholarship was his apprenticeship with the true giants of their time.

They exemplified true universal humanist principles (as I understand them):

- Be a humble and responsible servant to science, and to humanity!
- Always respect your fellow human, be it a student, a colleague, or a layperson!
- Don't abuse the privilege society has entrusted on you!
- Be always ready to learn from others and from your own mistakes!
- Look at human knowledge as a whole!

And so A. Mostowski adopted these principles from his teachers and tried to pass them on to the next generation.

## Macabre Memories of Zürich

This relief stands next to the entrance of the police department in Zürich, where A. Mostowski had to register as a foreign resident every few months.

How appropriate he thought: Three decapitated persons, holding their heads in their hands, decorate the entrance to the lion's den.

Well, they represent the martyrs Felix and Regula, and their servant, the city Saints of Zrich, early Christians who were decapitated by the Romans, and have no connections to the office in the building.



# Elitism vs Professionalism and High Standards

- Elitims today is based on exclusion rather than professional authority.
- Mathematics, and science in general, in Mostowski's time, was elitist, but elitisms was based, not on exclusion but on professional standards.
- In contrast to today, professional standards were very high, and there was a consensus about it.
- Students had to internalize these standars, and had to know by themselves whether they already had reached that level.

A. Mostowski treated his students as equal humans, but instilled in them the respect for professional standards.

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#### The Legacy, I: Two reports



### The Legacy, I: Foundational studies in 1955

A. Mostowski was mostly concerned with foundational studies:

A. Mostowski, The present state of investigations on the foundations of mathematics, Rozprawy Matemtyczne, vol. IX, Warsaw 1953.

The present stage of investigations on the foundation of mathematics opened at the time when the theory of sets was introduced. The abstractness of that theory and its departure from the traditional stock of notions which are accessible to experience, as well as the possibility of applying many of its results to concrete classical problems, made it necessary to analyze its epistemological foundations. This necessity became all the more urgent at the moment when antinomies were discovered. However, there is no doubt that the problem of establishing the foundations of the theory of sets would have been formulated and discussed even if no antinomy had appeared in the set theory.

## Thirty Years of Foundational Studies 1966

#### In 1964 he ends his lecture

A. Mostowski, Thirty Years of Foundational Studies, Lectures on the Development of Mathematical Logic and the Study of the Foundations of Mathematics in 1930-1964. Oxford 1966

as follows:

We stop here our presentation of what we consider as the most important results in the recent development of logic and the foundation of mathematics. The rate of development of these domains is presently so rapid that many new excellent results will certainly appear before these lectures will come to the hands of prospective readers. Let us hope that these new results will not only bring new interesting insights into the details but also allow us to form a sound judgement about outstanding problems in the philosophy of mathematics which have been waiting so long for a final solution.

# The Fragmentation of Mathematical Logic

The last monograph to cover all of set theory, recursion theory and model theory up to current research level was J. Shoenfield's Mathematical Logic published first in 1967.

By now the unified view of Mathematical Logic remains the privilege of the lucky few. For most of us Mathematical Logic may mean one of the following:

- A very specialized highly technical branch of pure mathematics: Model Theory, Set Theory, Computability Theory, with each of them fragmented into subdisciplines.
- An highly sophisticated engineering discipline, which uses logical tools to model situations in computer science, computer engineering and artificial intelligence with an eye on commercial applications.
- A branch of highly technical philosophy, with applications to epistemology, ethics, and the foundations of specialized reasoning.

I am afraid A. Mostowski would have found all these developments both intriguing and deplorable.

# The Legacy, II: Scientific Internationalisms

A. Mostowski was totally dedicated to *Science without borders*, and in the time of the Cold War, a main bridge builder for scientists on both sides of the Iron Curtain.

- **1959** Main initiator and organizer of the conference Infinitistic Methods in Warsaw, under the auspices of the International Mathematical Union and the Mathematical Institute of the Polish Academy of Sciences.
- **1964-1968** Vice president of the Section Logic, Methodology and Philosophy of Sciences, of the International Union of the History and Philosophy of Science.
- **1971-1975** President of the above.
- **1972** Co-initiator of the International Banach Center in Warsaw and director of the Logic Year at this center.

The 1959 and 1972 events in Warsaw were landmark events and left their impact beyond expectation.

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#### Infinitistic Methods 1959, Participants and Organizers

CSSR: L. Rieger (Prague) France: P. Février and D. Lacombe \* (Paris). R. Fraïssé (Alger), Germany: G. Asser (Berlin), P. Lorenzen (Hamburg). GB: G. Kreisel (Reading) Hungary: L. Kalm'ar (Szeged), R. Péter and J. Surányi (Budapest). Israel: **A. Robinson** (Jerusalem) Poland: A. Ehrenfeucht, A. Grzegorczyk, K. Kuratowski, A. Mostowski, H. Rasiowa, W. Sierpiński, R. Sikorski, M. Stark, A. Suliński and W. Zawadowski (Warsaw). D. Gierulanka (Cracow), S. Jaśkowski, J. Los and J. Slomiński (Toruń), M. Kokoszyńska-Lumanova, J. Mycielski and J. Slupecki (Wroclaw), The Netherlands: E. Beth and A. Heyting (Amsterdam) Roumania: Gr. Moisil (Bucarest). Sweden: S. Kanger (Uppsala). Switzerland: P. Bernays, E. Specker and G. Müller (Zürich) USA: J.W. Addison \* (Ann Arbor), W. Boone (Urbana), S.C. Kleene (Madison), S. MacLane and D. Scott (Chicago), R. Montague (Los Angeles), C. Spector (Columbus), L. Henkin, A. Tarski \* and R. L. Vaught (Berkeley). USSR: J.T. Medvedev \*, P.S. Novikov and A.J. Sragovič (Moscow). **Bold face**: Authors , \*: Speakers only. Colored: Organizing committee

Authors who did not attend:

R. MacDowell (Yellow Springs, USA), A.S. Essénine-Volpine (Moscow, USSR)

#### International long term visitors in Warsaw 1959-1974

We quote from S. Krajewski and M. Srebrny (2005)

A longer visit was thus, inter alia, received by:

J.W. Addison, M. Benda, M. Boffa, M. Dickmann,

- E. Frederikson, D. Giorgetta, P. Hinman, F.V. Jensen,
- R. Kowalski, E.G.K. Lopez-Escobar, M. Machover,
- J.A. Makowsky, K.C. Ng, K. Prikry, L. Rieger,
- H. Sayeki, Y. Suzuki, B.F. Wells, G. Wilmers,
- ... P. Hajek and P. Vopenka ...

and many more ... H. van Barendregt, e.g.

### The Legacy, III: Read and Disseminate

A true scholar reads and discusses other people's work.

- A. Mostwoski excelled in both. He
  - followed the developments of foundational studies,
  - discussed important papers in his seminars, and
  - wrote expository and explanatory papers to spread progress made by others.

Most remarkably on incompletenss results, undecidability and forcing.

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### The Legacy, IV: His most influential papers

A. Mostowski lived before the Publish or Perish craze. He neverthess has an H-index of approximately 25. His five most quoted works are:

[683 citations ] Set Theory, with an Introduction to Descriptive Set Theory, (with K. Kuratowski, edited and augmented by M. Maczyński and R. Kowalski), Warsaw 1976.

[543 citations ] On a generalization of quantifiers, FM 1957.

[133 citations ] Models of Axiomatic Theories Admitting Automorphisms, (with A. Ehrenfeucht), FM 1956

[101 citations ] Über die Unabhänigkeit des Wohlordnungssatzes vom Ordnungsprinzip, FM 1939.

[68 citations ] On direct products of theories, JSL 1952

# Work on Models of Arithmetic, Definability, and Hierarchies

The citation numbers do not reflect

- The impact of his expository writing,
- to what extent his publications were READ, and
- to what extent his papers triggered new lines of investigations.

In particular, this concerns his work on models of (also second order) arithmetic, definability of sets of natural numbers, and hierarchy theory.

# Andrzej Ehrenfeucht

![](_page_25_Picture_2.jpeg)

1932-

Mostowski's most visible and prolific Ph.D. student.

- Andrzej Ehrenfeucht, On Theories Categorical in Power, FM 1957
- Andrzej Ehrenfeucht, An Application of Games to the Completness Problem for Formalized Theories, FM 1960

Ehrenfeucht's paper with Mostowski, and the two papers cited on this slide mark the beginning of Classification Theory.

The work is further developed by R. Vaught and M. Morley, and then S. Shelah entered the world of model theory.

The rest is history!

# Per (Pelle) Lindström

![](_page_26_Picture_3.jpeg)

1936-2009

- Per Lindström, First Order Predicate Logic with Generalized Quantifiers Theoria, 1966,
- Per Lindström, On extensions of elementary logic, Theoria, 1969

It was A. Mostowski's paper on generalized quantifiers which inspired P. Lindström.

Lindström told me in 1980 that he really was looking for a new non-trivial application of the Ehrenfeucht-Fraïsse Theorem, which he had independentlay (re-)discovered by himself.

Thus Abstract Model Theory emerged and the book Model Theoreti Logics edited by S. Feferman and J. Barwise, testifies of its scope and impact.

## The Axiom of Choice

Following a suggestion of A. Lindenbaum to

#### extend and systematize the method

of A. Fränkel's proof of independence of the Axiom of choice, A. Mostowski had three fundamental papers on the Axiom of Choice:

- (with A. Lindenbaum)
  Über die Unabhängigkeit des Auswahlaxioms und einger seiner Folgerungen (1938)
- Über die Unabhänigkeit des Wohlordnungssatzes vom Ordnungsprinzip, FM 1939.
- Axiom of choice for finite sets, JSL 1948.

#### Nothing can shatter mathematical truth ...

In 1973, while I was in Warsaw, A. Mostowski came disturbed from a Ph.D. exam of a very gifted student.

How can it be, that the student failed so terribly, not even able to formulate the Fundamental Theorem of Algebra?

We suggested that he might have had a blackout.

Blackout ??!!??. If you wake me up in the middle of deep sleep, point a gun at me and ask me what is the Fundamental Theorem of Algebra, I would calmly tell you, and also prove it.

## Thomas Mann and Mathematics

Thomas Mann's wife Katya was the daughter of Alfred Pringsheim, a mathematician. She initially studied mathematics herself.

![](_page_29_Picture_4.jpeg)

Alfred Pringsheim 1850-1941

![](_page_29_Picture_6.jpeg)

![](_page_29_Picture_7.jpeg)

Katharina Mann 1883-1980, Thomas Mann 1875-1955

Thomas Mann was A. Mostowski's favorite author, and he read him in German. For Thomas Mann, and maybe also for A. Mostowski, mathematics was the

spiritual equivalent to passion.

I tell them that if they will occupy themselves with the study of mathematics they will find in it the best remedy against the lusts of the flesh. Thomas Mann (The Magic Mountain)

Warsaw, October 2013

# Thank you Panie Profesorze,

## thank you Professor Mostowski,

### for all you have taught us !

... and thank YOU those who are present for your attention !

#### Credits of photographs

• P. Lindström's picture was taken from the announcement of the University of Gothenburg of the The Lindstrm Lectures http://www.flov.gu.se/english/research/logic/lindstrom-lectures/

The picture is owned by P. Lindström's widow.

• A. Ehrenfeucht's picture was taken from Computer Science, University of Colorado Boulder

https://www.colorado.edu/cs/users/andrzej

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